A Mathematical Implementation of a “Global Human Walking Model with Real-Time Kinematic Personification” by Boulic, Thalmann and Thalmann.

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Abstract: The human motion algorithms from the paper "A Global Human Walking Model with Real-Time Kinematic Personification" by Boulic, Thalmann and Thalmann have been implemented in the Mathematica-programming language. The algorithms have been modified and extended to enable the motion of a 3-D stick figure. The velocity of 19 body parts (points on the stick figure) are computed as a function of time in the human walk cycle. Model predictions are visually consistent with actual human walking and Doppler acoustic sonar measurements made at the National Center for Physical Acoustics.

Discussion. The Boulic-Thalmann (BT) model (reference 1) is based upon experimental data. It is an empirical rather than dynamical model. It is based upon the motion of an average human being. It was developed to provide computationally efficient tools for aiding computer animation. Procedures are discussed in reference 1 for adding personification effects to predictions of human motion, but we do not consider these more complicated extensions in this brief paper. In the BT model, the body is represented as a stick figure. The components of motion which are addressed are: 1) pelvic roll, pitch and yaw, 2) flexion of the knee, hip and ankle, 3) yaw (or torsion) of the torso independent of the lower body (pelvis and legs), 4) flexion of the shoulder and elbow, and 5) three component translation of the body as a whole.

The coordinate system used in the BT model is shown in figure 1. The center of the system is the spine center. The z-axis is in the vertical direction and the x-axis points in the direction the individual is walking. The lower body is free to rotate in three directions (roll, pitch and yaw) about the spine center. The torso is free to rotate about the spine center independently of the lower body. Flexion occurs at the hip, knee and ankle for the right and left legs. Flexion occurs at the shoulder and elbow for the right and left arms. Left and right side leg and arm motions are symmetrical. Additionally, the body translates about spine center. In total there are twelve degrees of freedom.

Figure 1. Coordinate system for body motion used in the BT model.
Figure 2. Definition of key terms in the walk cycle.

One cycle of walking is illustrated in figure 2. The cycle consists of two steps. It begins with left foot heel strike (HS), continues through right foot HS, left foot toe off (TO) and ends with the second HS of the left foot. In the BT model, the velocity of walking is nondimensionalized in terms of thigh height. If $V$ denotes the velocity of walking in m/s, then $V$ is

$$V = RV \cdot H_{thigh},$$

where $RV$ is the relative velocity in units of “thigh height” per sec. Boulic and Thalmann consider relative velocities in the range $0 \leq RV \leq 2.3$. A brisk walk of about 4 mph corresponds to a relative velocity of about 1.7-1.8 depending upon thigh height of the individual.

In order to actually make predictions for a stick-figure body like the one shown in figure 1, we must have actual body dimensions. The author of this paper has a thigh height of one meter. His body dimensions (in inches) are shown in the accompanying table. These table values are used in the predictions to follow. Each measurement in the table is a triple of the form (x,y,z) with reference to the coordinate system in figure 1.

<table>
<thead>
<tr>
<th>Right</th>
<th>Left</th>
<th>Body</th>
</tr>
</thead>
<tbody>
<tr>
<td>hip (0,-7,-7)</td>
<td>hip (0,7,-7)</td>
<td>spine center (0,0,0)</td>
</tr>
<tr>
<td>knee (0,-7,25.37)</td>
<td>knee (0,7,25.37)</td>
<td>throat bottom (0,0,11.63)</td>
</tr>
<tr>
<td>ankle (0,-7,-46.37)</td>
<td>ankle (0,7,-46.37)</td>
<td>throat top (0,0,15.63)</td>
</tr>
<tr>
<td>toe (8,-7,-46.37)</td>
<td>toe (8,7,-46.37)</td>
<td>head center (0,0,20.67)</td>
</tr>
<tr>
<td>shoulder (0,-8.5,11.63)</td>
<td>shoulder (0,8.5,11.63)</td>
<td>head top (0,0,25.67)</td>
</tr>
<tr>
<td>elbow (0,-8.5,-0.37)</td>
<td>elbow (0,8.5,-0.37)</td>
<td></td>
</tr>
<tr>
<td>fingers (0,-8.5,-17.37)</td>
<td>fingers (0,8.5,17.37)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Body dimensions in inches for an individual with a one meter thigh height.
In order to compute the position of the critical body parts shown in table 1, we will make use of the fact that the motion of a rigid body can be described by applying an appropriate set of rotations and translations. A set of parameters for doing this is described in reference 1. We will first consider motion of the lower body parts. This requires a four-step process. First, we consider the effect of hip flexion on the position of the knee, ankle and toe. If $\theta_{\text{hip}}$ denotes the angle of the hip at some instant of time, then the knee, ankle and toe in relation to the hip will be at the vector locations

$$\mathbf{X}_{\text{knee},1} = M_y[\theta_{\text{hip}}](\mathbf{X}_{\text{knee},0} - \mathbf{X}_{\text{hip},0}) + \mathbf{X}_{\text{hip},0}$$

$$\mathbf{X}_{\text{ankle},1} = M_y[\theta_{\text{hip}}](\mathbf{X}_{\text{ankle},0} - \mathbf{X}_{\text{hip},0}) + \mathbf{X}_{\text{hip},0}$$

$$\mathbf{X}_{\text{toe},1} = M_y[\theta_{\text{hip}}](\mathbf{X}_{\text{toe},0} - \mathbf{X}_{\text{hip},0}) + \mathbf{X}_{\text{hip},0}$$

where $M_y[\theta_{\text{hip}}]$ is the rotation matrix about the y-axis that describes the hip flexion:

$$M_y[\theta_{\text{hip}}] = \begin{bmatrix} \cos \theta_{\text{hip}} & 0 & -\sin \theta_{\text{hip}} \\ 0 & 1 & 0 \\ \sin \theta_{\text{hip}} & 0 & \cos \theta_{\text{hip}} \end{bmatrix}$$

Second, if $\theta_{\text{knee}}$ denotes the angle of the knee at some instant, the ankle and toe in relation to the knee will be at the vector locations

$$\mathbf{X}_{\text{ankle},2} = M_y[\theta_{\text{knee}}](\mathbf{X}_{\text{ankle},1} - \mathbf{X}_{\text{knee},1}) + \mathbf{X}_{\text{knee},1}$$

$$\mathbf{X}_{\text{toe},2} = M_y[\theta_{\text{knee}}](\mathbf{X}_{\text{toe},1} - \mathbf{X}_{\text{knee},1}) + \mathbf{X}_{\text{knee},1}$$

where $M_y[\theta_{\text{knee}}]$ is the rotation matrix about the y-axis that describes the knee flexion. Third, if $\theta_{\text{ankle}}$ denotes the angle of the ankle at some instant, the toe in relation to the ankle will be at the vector location

$$\mathbf{X}_{\text{toe},3} = M_y[\theta_{\text{ankle}}](\mathbf{X}_{\text{toe},2} - \mathbf{X}_{\text{ankle},2}) + \mathbf{X}_{\text{ankle},2}$$

where $M_y[\theta_{\text{ankle}}]$ is the rotation matrix about the y-axis that describes the ankle flexion. Finally, three dimensional rotation of the pelvis about the spine center through roll, pitch and yaw angles and translation of the pelvis will cause the hip, knee, ankle and toe to move to the final locations

$$\mathbf{X}_{\text{hip}} = M_{xyz}[\phi, \theta, \psi] \mathbf{X}_{\text{hip},0} + \mathbf{X}_{\text{pelvis}}$$

$$\mathbf{X}_{\text{knee}} = M_{xyz}[\phi, \theta, \psi] \mathbf{X}_{\text{knee},1} + \mathbf{X}_{\text{pelvis}}$$

$$\mathbf{X}_{\text{ankle}} = M_{xyz}[\phi, \theta, \psi] \mathbf{X}_{\text{ankle},2} + \mathbf{X}_{\text{pelvis}}$$

$$\mathbf{X}_{\text{toe}} = M_{xyz}[\phi, \theta, \psi] \mathbf{X}_{\text{toe},3} + \mathbf{X}_{\text{pelvis}}$$

where

$$M_{xyz}[\phi, \theta, \psi] = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ -\sin \phi & 0 & \cos \phi \end{bmatrix}$$
Motion of the shoulder, elbow and fingers are dealt with in a similar fashion. Flexion of the shoulder will cause the elbow and fingers to move with respect to the shoulder in the following way.

\[ \mathbf{X}_{\text{elbow},1} = M_y [\theta_{\text{shoulder}}] (\mathbf{X}_{\text{elbow},0} - \mathbf{X}_{\text{shoulder},0}) + \mathbf{X}_{\text{shoulder},0} \]

\[ \mathbf{X}_{\text{finger},1} = M_y [\theta_{\text{shoulder}}] (\mathbf{X}_{\text{finger},0} - \mathbf{X}_{\text{shoulder},0}) + \mathbf{X}_{\text{shoulder},0} \]

Flexion of the elbow will produce the following movement of the fingers relative to the elbow:

\[ \mathbf{X}_{\text{finger},2} = M_y [\theta_{\text{elbow}}] (\mathbf{X}_{\text{finger},1} - \mathbf{X}_{\text{elbow},1}) + \mathbf{X}_{\text{elbow},1} \]

Rotation of the torso about the spine center and translation of the body as a whole will cause the shoulders, elbow and fingers to move to the vector locations

\[ \mathbf{X}_{\text{shoulder}} = M_z [\psi_{ub}] \mathbf{X}_{\text{shoulder},0} + \mathbf{X}_{\text{pelvis}} \]

\[ \mathbf{X}_{\text{elbow}} = M_z [\psi_{ub}] \mathbf{X}_{\text{elbow},1} + \mathbf{X}_{\text{pelvis}} \]

\[ \mathbf{X}_{\text{finger}} = M_z [\psi_{ub}] \mathbf{X}_{\text{finger},2} + \mathbf{X}_{\text{pelvis}} \]

where rotation of the torso about the spine center is described by the matrix

\[ M_z [\psi_{ub}] = \begin{bmatrix} \cos \psi_{ub} & \sin \psi_{ub} & 0 \\ -\sin \psi_{ub} & \cos \psi_{ub} & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

Figure 3 shows predictions of the 9 angles and 3 displacements used in the preceding equations. The relative velocity is \( RV = 1 \). These graphs were computed using the empirical equations developed by Boulic and Thalmann (reference 1). In each plot in figure 3, the horizontal axis is relative time. A step with the left and right foot requires a relative time interval of length one.

The BT model makes predictions in terms of relative time. In order to compute body part velocities we must operate in terms of absolute time. To this end we must consider two dimensional quantities. The most important spatial characteristic of a walk cycle is the relative cycle length defined by \( RL_c = 1.346 \sqrt{RV} \). The fundamental temporal characteristic of a walk cycle is the duration \( D_c = RL_c / RV \). The relative time in a walk cycle is \( \tau = t / D_c \) where \( t \) is absolute time and \( \tau \) is relative time. If \( \mathbf{X}(\tau, RV) \) denotes the position of a body part at relative time \( \tau \), then the velocity of this body part can be shown with the aid of the chain rule to be

\[ \mathbf{v}(\tau, RV) = H_{\text{thigh}} \frac{RV}{RL_c} \frac{\partial}{\partial \tau} \mathbf{X}(\tau, RV). \]

This last equation has been used to compute the graphs of body part velocities shown in figure 4. The derivative has been computed using numerical differentiation. The upper portion of the figure shows the velocities of the left hip, knee, ankle and toe (tips). The middle portion of the figure shows the velocities of the left shoulder, elbow and finger (tips). The lower portion of the figure shows the velocities for each body part in table 1. Left-side and right-side arm and leg motion are one-half cycle out of phase. All computations in figure 4 are for a relative velocity \( RV = 1 \) and unit thigh height.
Figure 5 shows a comparison of measured micro-Doppler data and predictions of body part velocities appropriate for the experimental geometry. The measured data were recorded at the National Center for Physical Acoustics (NCPA). A continuous-wave Doppler sonar operating at a carrier frequency of 40 kHz was used to make the measurements. Data were recorded using 24 bit technology at a sample rate of 96 kHz. The recorded data shown in the figure are for a time period that is 1.7 sec long during which an individual with a height of 67 inches was walking away from the sonar. The color plot on the left hand side of figure 5 is a short-time spectrogram. The dynamic range in the plot is 20 dB. An FFT of size 8192 with a Hamming window has been used to reveal the individual Doppler components in the data. Frequency has been converted to velocity of body part by applying the Doppler formula \[ \Delta f = \frac{2v}{\lambda} \] where \( \Delta f \) is the frequency of the appropriate FFT bin relative to 40 kHz, \( \lambda \) is acoustic wavelength (nominally 0.8275 cm) and \( v \) is velocity. The bright red line in the measured spectrogram at a velocity of 0 m/s is the carrier frequency. The right hand side of figure 5 shows model predictions of body part velocities using the procedures described in this paper. The thigh height of the individual has been estimated to be 0.93 m. His walking velocity has been estimated from the measured data. The visual agreement between the measured and modeled results is quite good. The measurements show velocities of about -4 m/s corresponding to similar model predictions for velocities of the ankle-toe complex. These are velocity components in the measured data that are consistent with the velocities of the torso and pelvis. It is quite striking that so much of the character in the data can be predicted with a model with only 12 degrees of freedom.

**Conclusions.** The BT model produces motions that are visually correct. This can be seen by looking at movie loops of the motions of stick figure models computed using the equations in this paper. Predictions of body part velocities are in qualitative agreement with measured Doppler-sonar data.

**References**
Figure 3. Angles and displacements for the BT for relative velocity $RV = 1$. 
Figure 4. Horizontal velocity of the left hip, knee, ankle and toes (upper portion), horizontal velocity of the left shoulder, elbow and finger tips (middle) and all body parts (lower).
Figure 5. Comparison of measured and modeled Doppler data.